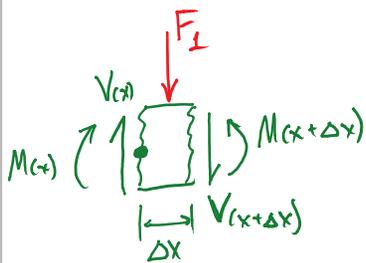
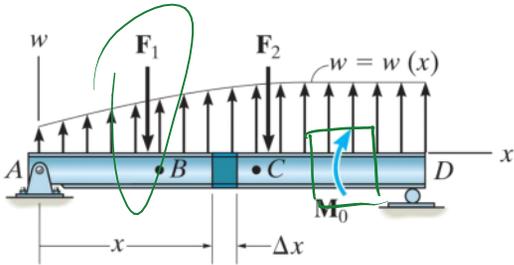


# Relations Among Load, Shear and Bending Moments



$$\sum F_y = 0 \Rightarrow V(x) - V(x+\Delta x) - F_1 = 0$$

$$V(x+\Delta x) - V(x) = -F_1$$

take  $\Delta x \rightarrow 0$

$\Rightarrow$  step in  $V(x) = |F_1|$  down

$$(\sum M)_x = 0$$

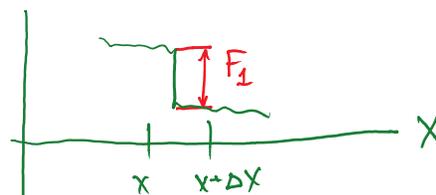
$$\Rightarrow -M(x) + M(x+\Delta x) - F_1 \cdot \frac{\Delta x}{2} - V(x+\Delta x) \cdot \Delta x = 0$$

$$M(x+\Delta x) - M(x) = F_1 \cdot \frac{\Delta x}{2} - V(x+\Delta x) \Delta x \xrightarrow{0} 0$$

take  $\Delta x \rightarrow 0$

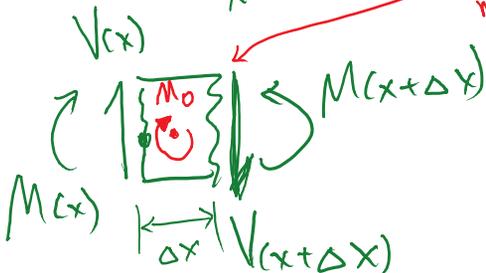
$$\Rightarrow M(x+\Delta x) - M(x) = 0$$

**No step!**



$M_0$  is between  $x$  and  $(x+\Delta x)$

the couple has magnitude  $M_0$ , but is in  $-\hat{k}$  direction



$$(\sum M)_x = 0$$

$$-M(x) + M(x+\Delta x) - M_0 - \Delta x \cdot V(x+\Delta x) = 0$$

$$M(x+\Delta x) - M(x) = M_0 + \Delta x \cdot V(x+\Delta x)$$

value of  $M_0$   $\rightarrow 0$   $\Rightarrow$   $V \rightarrow 0$   $\Rightarrow$   $M \rightarrow 0$



$$\sum F_y = 0$$

$$V(x) - V(x+\Delta x) = 0$$

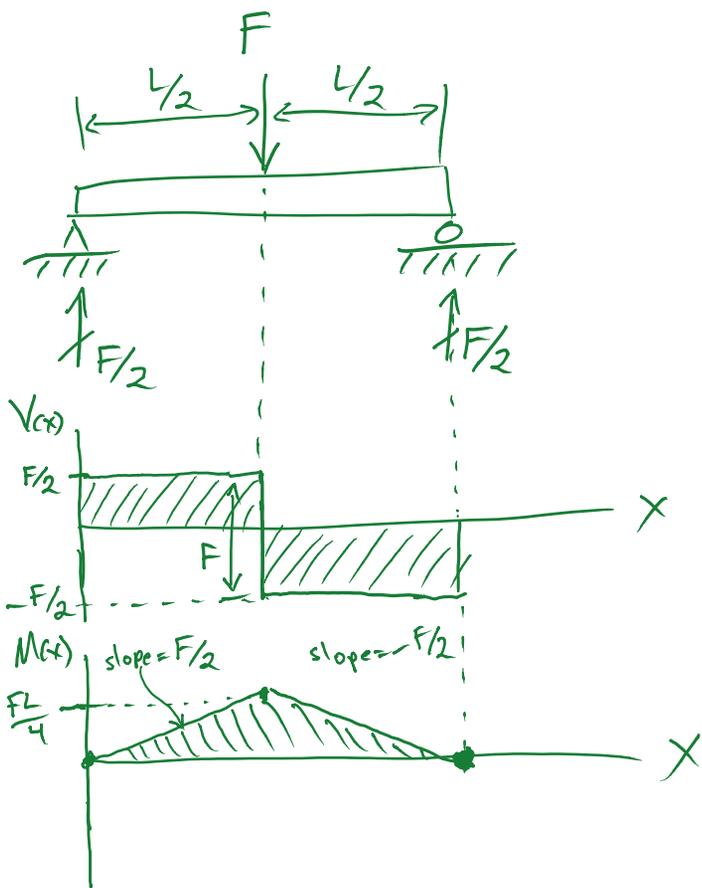
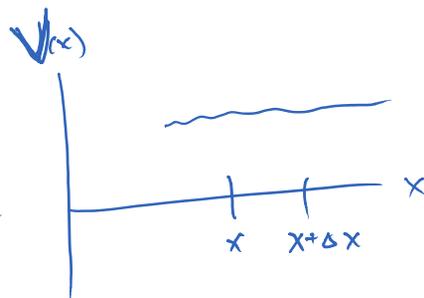
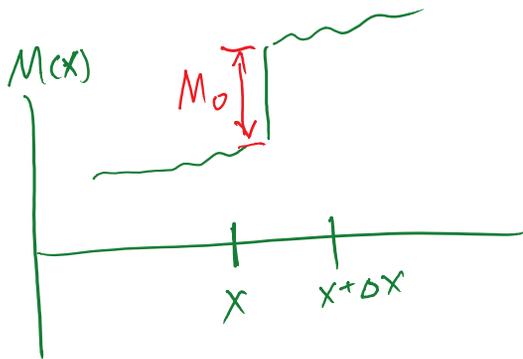
$$\Rightarrow V(x+\Delta x) = V(x)$$

$\Rightarrow$  No step in  $V(x)$

Value of step take  $\Delta x \rightarrow 0$

$$\Rightarrow \boxed{M(x+\Delta x) - M(x) = M_0}$$

$\Delta x \rightarrow 0$   
step is upward!



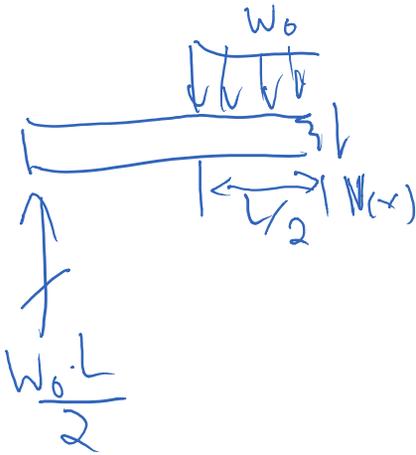
$$\boxed{\frac{dM}{dx} = V(x)!}$$

# Remember

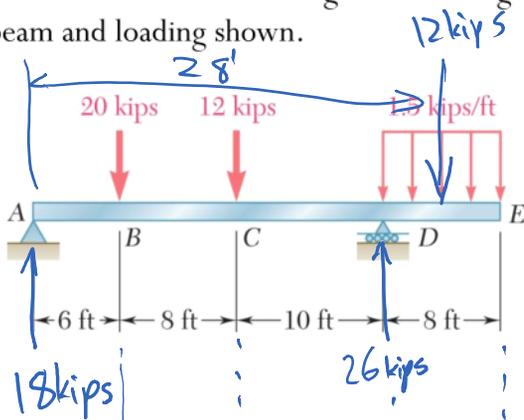
$$\frac{dM}{dx} = V(x)$$

$V(x)$  gives slope  
of  $M(x)$

$$\frac{dV}{dx} = \underbrace{w(x)}_{\text{distributed load}}$$



Draw the shear and bending moment diagrams for the beam and loading shown.



$$\sum M_A = 0$$

$$-120 \text{ kip}\cdot\text{ft} - 168 \text{ kip}\cdot\text{ft} + 24 D_y - (28')(12 \text{ kips}) = 0$$

$$(-288 - 336) = -24 D_y$$

$$D_y = \frac{624 \text{ kips}}{24}$$

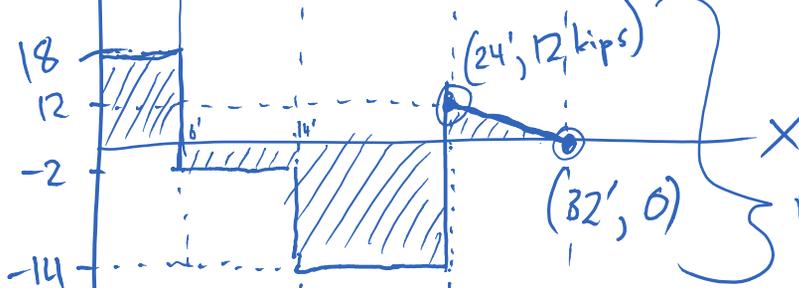
$$D_y = 26 \text{ kips}$$

$$\sum F_y = A_y + D_y - 20 \text{ kips} - 12 \text{ kips} - 12 \text{ kips} = 0$$

$$A_y + 26 \text{ kips} - 44 \text{ kips} = 0$$

$$A_y = 18 \text{ kips}$$

$V(x)$   
(kips)



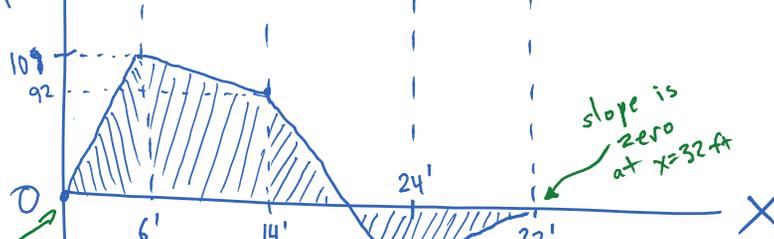
plug into  
 $y = mx + b$  form

$$V(x) = m \cdot x + b$$

solve  $m$  &  $b$

$$m = -1.5 \text{ kips/ft}$$

$M(x)$   
(kip-ft)



see that  $\frac{dM}{dx} = V(x)$

$M(0) = 0$   
because no couple at  $x = 0$

slope is 18 kips  
slope is -2 kips  
slope is -14 kips  
slope is 12 kips at  $x = 24'$  ft  
concave down

### Cantilever beam with distributed load

Consider a cantilever beam subjected to a uniform distributed load  $w$  and a point load  $P = wL/2$  as indicated below.

Draw the free-body diagram and corresponding shear force and bending moment diagrams.

To draw the shear force and bending moment diagrams, you MUST use the minimum number of lines (straight or curved), i.e., the minimum number of objects created by clicking the two buttons under "V and M lines".

**FBD**

**V(x)**

**M(x)**

**FBD Concentrated forces:**

↑ ↓ → ←

**FBD Distributed loads:**

↑↑↑ ↓↓↓

↑↑↑ ↓↓↓

**FBD Moments:**

↷ ↶

**V and M lines:**

↷ ↶

**Help buttons (not graded):**

⋮ ✕

*Handwritten notes:*

- If reaction in x is zero, PrairieLearn will allow you to omit the vector.
- Reaction couple should be positive. Call it  $M_0$ .
- $V(L) = 0$  because no point load at end of beam.
- $M(L) = 0$  because no couple at end of beam.
- $M(x) < 0$  everywhere!
- "Frowning" beam
- $M(x)$  concave down here
- slope should be continuous here, but PrairieLearn is not picky about that
- $M(0) = -M_0$  because positive couples cause a downward step

Tolerance for placement of objects is 1/2 grid size

Submit New variant

### SSB with distributed load

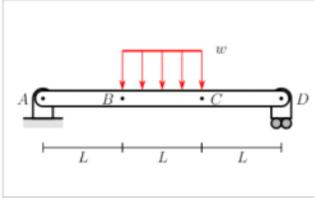
Consider a simply-supported beam subjected to a uniform distributed load  $w$  as indicated below.

Draw the free-body diagram and corresponding shear force and bending moment diagrams.

To draw the shear force and bending moment diagrams, you MUST use the minimum number of lines (straight or curved), i.e., the minimum number of objects created by clicking the two buttons under "V and M lines".

SSB with distributed load

Consider a simply-supported beam subjected to a uniform distributed load  $w$  as indicated below.



Draw the free-body diagram and corresponding shear force and bending moment diagrams.

To draw the shear force and bending moment diagrams, you MUST use the minimum number of lines (straight or curved), i.e., the minimum number of objects created by clicking the two buttons under "V and M lines".

FBD

FBD Concentrated forces:

↑ ↓ → ←

FBD Distributed loads:

↑↑↑ ↓↓↓

↑↑ ↓↓ ↑↑ ↓↓

FBD Moments:

⤵ ⤶

V and M lines:

⤵ ⤶

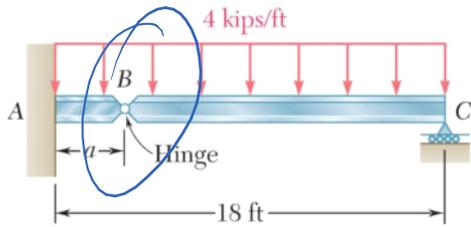
Help buttons (not graded):

⋮ ✕

Tolerance for placement of objects is 1/2 grid size

Submit New variant

Draw the shear and bending moment diagrams for the beam and loading shown.



At a hinge, what are  $M$  &  $\Delta V$ ?

- A)  $M > 0$
- B)  $M < 0$
- C)  $M$  unknown

D)  $M = 0$

$\Delta V$ ? (step in  $V(x)$ )

A) 0

B)  $V < 0$

C)  $V > 0$

D)  $V = ?$  (could be either?)